Sample Spaces

Tossing a coin four times is an example of a probability experiment. Below are listed the sixteen possible outcomes.

HHHH	THHH
HHHT	THHT
HHTH	THTH
HHTT	THTT
HTHH	TTHH
HTHT	TTHT
HTTH	TTTH
HTTT	TTTT

Any set of outcomes for a probability experiment in which one of the outcomes must happen and not more than one can happen at the same time is called a sample space for the experiment.

The sixteen outcomes above is a sample space for the tossing of a coin four times experiment.

Each outcome in a sample space is called a sample point.

A subset of a sample space is called an event. For example, the event that the first two tosses are heads is an event whose sample points are: HHHH, HHHT, HHTH, and HHTT.

Probabilities

Each sample point is assigned a number called a probability. This number cannot be negative or greater than one. The sum of the probabilities for all the sample points must equal one.

The probability of an event is the sum of the probabilities of it's sample points. For example, in the coin tossing experiment each sample point can be assigned the probability 1/16 so they all add up to one and then the probability of the event that the first two tosses are heads would be $1/16 + 1/16 + 1/16 + 1/16 = 1/4$.

The set of all sample points that belong to the events A_1 , A_2 , . ,A_N is called the union of the events A_1 , A_2 , . ,A_N . The probability that one of the events occurs is the probability of their union.

Addition Rule

If two or more events have no sample points in common, the probability that one of them will occur is equal to the sum of their probabilities.
If two of them had a sample point in common then in the sum of their probabilities, the probability of the sample point in common would be added twice thus overstating the probability of the union of the events.

Compound Events

The event that events A and B both occur is called a compound event and is denoted by AB and it's probability is denoted by P(AB).

Conditional Probability

The conditional probability that event A occurs given that event B occurred is denoted by P(A|B) .   .

$$P(A|B) = \frac{P(AB)}{P(B)}$$

For example, assuming all the sample points have the same probability and that there are n sample points, then if event A has two sample points in common with event B and event B has four sample points, then since we know one of the four sample points from event B occurred and that A has two of those points then P(A|B) = 2/4 .
So since P(AB) = 2/n   and P(B) = 4/n , P(AB)/P(B) = 2/n / 4/n = 2/4, so $P(A|B) = \frac{P(AB)}{P(B)}$.

Multiplication rule for compound events

P(AB) =P(A|B)P(B) .

Independent events

If P(A) = P(A|B) then the knowledge that event B occurred does not change the probability of event A occurring. So, we say the events are independent. So P(A) = P(AB)/P(B). So P(AB) = P(A)P(B).

Multiplication rule for independent events

$P(AB) = P(A)P(B)$

Definition of independent events

Two events A and B are independent if $P(AB) = P(A)P(B)$.

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